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Mass spectrum of $\mathcal{N} = 8$ supergravity on $AdS_2 \times S^2$

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Abstract: An initial step is taken in investigating the duality between the near horizon region of a four dimensional extremal Reissner-Nordström black hole and the n-particle, $\mathcal{N} = 4$ Calogero model as conjectured by Gibbons and Townsend. Specifically we compute the mass spectrum of $d = 4$, $\mathcal{N} = 8$ supergravity about the Bertotti-Robinson solution and find the corresponding set of conformal dimensions of states in the dual conformal quantum mechanics. We find that the dual states fill irreducible representations of the supergroup $\mathrm{SU}(1,1|2)$, and furthermore transform under various irreducible representations of the group $\mathrm{SU}(2) \times \mathrm{SU}(6)$ spontaneously broken down from the $E_{7(7)}$ duality group of $\mathcal{N} = 8$ supergravity.

Keywords: D-branes, Black Holes in String Theory, Supergravity Models.

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Contents

1. Introduction

The AdS_2/CFT_1 duality conjectured by Maldacena [1] has received relativity little attention as compared to some of it's higher dimensional cousins. It is however one of the most interesting cases with regard to black holes as the geometry $AdS_2 \times S^2$ arises as the near horizon geometry of a Reissner-Nordström black hole. While the higher dimensional AdS/CFT dualities have been primarily used to learn about gauge theory through gravity, one hopes that the reverse can be done for four-dimensional black holes. That is to say, it seems natural that a conformal quantum mechanics will be simpler than a supergravity theory on AdS_2. Some recent investigations into the AdS_2/CFT_1 duality are given in [2, 3].

Since the AdS/CFT duality was discovered by studying D-brane configurations, in the hope of applying the duality to 4 dimensional black holes it is reasonable to begin by looking for such solutions which have a D-brane interpretation. One such solution [4] consists of four sets of D-branes in which any pair intersects over a string. Wrapping the D-branes on a six-torus and taking each set of the four intersecting D-branes to have equal charge gives rise to the extreme Reissner-Nordström solution. The near horizon geometry of this solution is well known to be $AdS_2 \times S^2$, with isometry supergroup $\mathrm{SU}(1,1|2)$. The results of [5] further indicate that in fact $AdS_2 \times S^2$ is a solution to the full type IIB string theory and therefore that $\mathrm{SU}(1,1|2)$ must also be a symmetry of the dual conformal quantum mechanics. Gibbons and Townsend [3] have conjectured that the dual CFT is given by the n-particle, $\mathcal{N}=4$ superconformal Calogero model, which has yet to be constructed for arbitrary n (see [6] for the $n=2$ case.)

As a first step toward investigating this conjecture we consider the AdS_2 side of the duality. The low energy limit of type II(A or B) string theory on T^6 is the $\mathcal{N}=8$ supergravity theory of Cremmer and Julia [7]. Off-shell this theory has an SO(8) symmetry while on-shell the symmetry is enhanced to an $E_{7(7)}$ duality. We expand this theory about the above D3-brane solution in the near horizon limit to linearized order in the fluctuations to find the mass spectrum. Using the prescription of Gubser, Klebanov, and Polyakov [8] and Witten [9] we then extract the conformal weights of the dual CFT states and show that they lie in irreducible representations of $\mathrm{SU}(1,1|2)$, consistent with the fact that six of the supersymmetries are broken in the near horizon limit of the D3-brane solution [10]. Furthermore the $E_{7(7)}$ duality symmetry is broken to $\mathrm{SU}(2) \times \mathrm{SU}(6)$ [11] and indeed the dual CFT fields lie in irreducible representations of this group as well.

The procedure for extracting the masses is well known. For example, type IIB supergravity on $AdS_5 \times S^5$ was considered in [12, 13] and 11 dimensional supergravity on $AdS_4 \times S^7$ and $AdS_7 \times S^4$ (among other spacetimes) in [14]. More recently the mass spectrum of 6 dimensional, $N=4b$ supergravity on $AdS_3 \times S^3$ [15] was found for which the dual CFT is 2 dimensional and detailed checks on the correctness of the conjectured duality could be made, see [16] and references therein. Other cases considered recently include 5 dimensional simple supergravity on $AdS_3 \times S^2$ [17] and $AdS_2 \times S^3$ [18], and $\mathcal{N}=8$ supergravity on $AdS_3 \times S^2$ [11]. In this paper we follow closely the procedure of [12] to compute the mass spectrum of 4 dimensional $\mathcal{N}=8$ supergravity about $AdS_2 \times S^2$.

In section 2 we briefly describe the $d=4$, $\mathcal{N}=8$ supergravity theory of Cremmer and Julia [7] and give the Bertotti-Robinson (BR) solution [19] consisting of an $AdS_2 \times S^2$ geometry with a nonvanishing two-form flux on the two-sphere. In section 3 we diagonalize the bosonic equations of motion to obtain their mass spectrum, and repeat the procedure in section 4 for the fermionic fields. In section 5 we go on to compute the conformal dimensions of the corresponding states of the conformal quantum mechanics model and demonstrate that they lie in irreducible representations of $\mathrm{SU}(1,1|2)$. In section 6 we end with some conclusions.

We work in the signature $(-,+,+,+)$.

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2. $\mathcal{N} = 8$ supergravity

2.1 Field content

The field content of $d = 4$, $\mathcal{N} = 8$ supergravity consists of a graviton described by a vierbein[1] $e_{\hat\mu}{}^a$, 8 gravitinos described by the Majorana Rarita-Schwinger fields $\psi_{\hat\mu A}$ for $A = 1, \ldots, 8$, 28 vector fields described by the Abelian gauge fields $B_{\hat\mu}^{MN}$ (antisymmetric in M, N) for $M, N = 1, \ldots, 8$, 56 spin-1/2 fields described by the Majorana spinors λ_{ABC} (antisymmetric in A, B, C) for $A, B, C = 1, \ldots, 8$, and 70 spin-0 fields desribed by the scalar fields W_{ABCD}. The $O(8)$ invariant lagrangean density describing this theory is given by

$$
\begin{aligned}
\mathcal{L} = {} & \frac{1}{4} e R(\omega, e) + \frac{1}{2} \epsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma} \bar\psi_{\hat\mu A} \gamma_{\hat\sigma} \gamma_5 \left(\delta_A{}^B D_{\hat\nu}(\omega) - Q_{\hat\nu A}{}^B \right) \psi_{\hat\rho B} + \\
& + \frac{1}{8} e G^{MN}_{\hat\mu\hat\nu}(B) \tilde H^{\hat\mu\hat\nu}_{MN}(B, \mathcal{V}, \psi, \lambda) - \frac{1}{12} i e \bar\lambda_{ABC} \gamma^{\hat\mu} \left(\delta_A{}^D D_{\hat\mu}(\omega) - 3 Q_{\hat\mu A}{}^D \right) \lambda_{BCD} - \\
& - \frac{1}{24} e P_{\hat\mu ABCD} \bar P^{\hat\mu ABCD} - i \frac{1}{6\sqrt{2}} \bar\psi_{\hat\mu A} \gamma^{\hat\nu} \gamma^{\hat\mu} \left(\bar P_{\hat\nu}^{ABCD} + \hat{\bar P}_{\hat\nu}^{ABCD} \right) \lambda_{BCD} + \\
& + \frac{1}{8\sqrt{2}} \Bigg(i \bar\psi_{\hat\nu A} \gamma^{[\hat\nu} \hat{\mathcal{F}}_{AB} \gamma^{\hat\mu]} \psi_{\hat\nu B} - \frac{1}{\sqrt{2}} i \bar\psi_{\hat\mu C} \hat{\mathcal{F}}_{AB} \gamma^{\hat\mu} \lambda_{ABC} + \\
& \qquad + \frac{i}{72} \eta \, \epsilon^{ABCDEFGH} \bar\lambda_{ABC} \hat{\mathcal{F}}_{DE} \lambda_{FGH} \Bigg) ,
\end{aligned}
\tag{2.1}
$$

where

$$
\begin{aligned}
G^{MN}_{\hat\mu\hat\nu} &= 2 \partial_{[\hat\mu} B^{MN}_{\hat\nu]} \\
\hat P_{\hat\mu ABCD} &= P_{\hat\mu ABCD} + i 2\sqrt{2} \left(\bar\psi^{(L)}_{\hat\mu [A} \lambda^{(R)}_{BCD]} + \frac{1}{24} \eta \, \epsilon_{ABCDEFGH} \bar\psi_{\hat\mu}^{(R)E} \lambda^{(L)FGH} \right) \\
\hat{\mathcal{F}}_{AB} &= \gamma^{\hat\mu\hat\nu} \hat{\mathcal{F}}_{AB\hat\mu\hat\nu} \\
\hat{\mathcal{F}}_{AB\hat\mu\hat\nu} &= \mathcal{F}^{(F)}_{AB\hat\mu\hat\nu} + \sqrt{2} \Bigg(i \bar\psi^{(R)}_{[\hat\mu[A} \psi^{(L)}_{\hat\nu]B]} - i \frac{1}{\sqrt{2}} \bar\psi^{(L)C}_{[\hat\mu} \gamma_{\hat\nu]} \lambda^{(L)}_{ABC} + \\
& \qquad + \frac{i}{288} \eta \, \epsilon_{ABCDEFGH} \bar\lambda^{CDE}_{(L)} \gamma_{\hat\mu\hat\nu} \lambda^{FGH}_{(R)} \Bigg) \\
D_{\hat\mu}(\omega) \lambda &= \left(\partial_{\hat\mu} + \frac{1}{4} \omega^{ab}_{\hat\mu} \gamma_{ab} \right) \lambda \, .
\end{aligned}
\tag{2.2}
$$

The $(R), (L)$ superscripts on the fermions denote right and left-handed components defined by $\lambda^{(R/L)} := (1/2)(1 \pm \gamma_5)\lambda$ (γ-matrix conventions are described in the appendix). $\mathcal{F}^{(F)}_{AB\hat\mu\hat\nu}$ and $\tilde H^{\hat\mu\hat\nu}_{MN}$ are defined below.

[1] Hatted Greek indices are used for 4D coordinate indices whereas unhatted Greek indices denote 2D coordinate indices with $\mu, \nu, \ldots = 0,1$ and $\alpha, \beta, \ldots = 2,3$.

The action possesses an SU(8) gauge symmetry with gauge field $Q_{\hat{\mu}A}{}^{B}$. Capital Latin letters $A, B, C, \ldots$ denote SU(8) indices with lower(upper) indices transforming in the $8(\bar{8})$ representation. When an infinitesimal SU(8) transformation $\Lambda_A{}^B = \Lambda'_A{}^B + i\Lambda''_A{}^B$ (where $\Lambda'_A{}^B = -\Lambda'_B{}^A$ and $\Lambda''_A{}^B = \Lambda''_B{}^A$) acts on a Majorana-Fermi field the i is replaced by $i\gamma_5$ to preserve the Majorana condition. This also explains why some contracted SU(8) indices in the action (2.1) are both in the lower position.

The scalars $P_{\hat{\mu}ABCD}$ and SU(8) gauge fields $Q_{\hat{\mu}A}{}^{B}$ can be grouped together into an element of $E_{7(7)}$ in the fundamental 56 representation as

$$\partial_{\hat{\mu}} \mathcal{V} \mathcal{V}^{-1} = \begin{pmatrix} Q_{\hat{\mu}[A}{}^{[C} \delta_{B]}{}^{D]} & P_{\hat{\mu}ABCD} \\ \bar{P}_{\hat{\mu}}{}^{ABCD} & \bar{Q}_{\hat{\mu}}{}^{[A}{}_{[C} \delta^{B]}{}_{D]} \end{pmatrix}, \tag{2.3}$$

where $\bar{Q}_{\hat{\mu}}{}^{B}{}_{A} := (Q_{\hat{\mu}A}{}^{B})^*$, $\bar{P}_{\hat{\mu}}{}^{ABCD} := (P_{\hat{\mu}ABCD})^*$, and $P_{\hat{\mu}ABCD}$ satisfies the constraint

$$P_{\hat{\mu}ABCD} = \frac{1}{24} \eta\, \epsilon_{ABCDEFGH} \bar{P}_{\hat{\mu}}{}^{EFGH}, \tag{2.4}$$

where $\eta = \pm 1$. The field $\mathcal{V}$ in this sense is analogous to the vierbein $e_{\hat{\mu}}{}^{a}$ and tranforms under SU(8) on the left and $E_{7(7)}$ on the right.

On shell the theory also possesses an $E_{7(7)}$ symmetry acting on the right of $\mathcal{V}$ and on the left of the column vector

$$\begin{pmatrix} \mathcal{F}^{(F)}_{\hat{\mu}\hat{\nu}MN} \\ \bar{\mathcal{F}}^{(F)MN}_{\hat{\mu}\hat{\nu}} \end{pmatrix} := \frac{1}{\sqrt{2}} \begin{pmatrix} G^{MN}_{\hat{\mu}\hat{\nu}} + iH_{\hat{\mu}\hat{\nu}MN} \\ G^{MN}_{\hat{\mu}\hat{\nu}} - iH_{\hat{\mu}\hat{\nu}MN} \end{pmatrix}, \tag{2.5}$$

where the M, N indices are $E_{7(7)}$ indices. For the vector field the $E_{7(7)}$ transformation simply mixes the equations of motion and Bianchi identities given by

$$\nabla^{\hat{\mu}} \left(\tilde{\mathcal{F}}^{(F)MN}_{\hat{\mu}\hat{\nu}} \mp \tilde{\bar{\mathcal{F}}}^{(F)MN}_{\hat{\mu}\hat{\nu}} \right) = 0 \tag{2.6}$$

respectively.

To convert $E_{7(7)}$ indices to SU(8) indices one acts with $\mathcal{V}$, eg.,

$$\begin{pmatrix} \mathcal{F}^{(F)}_{\hat{\mu}\hat{\nu}AB} \\ \bar{\mathcal{F}}^{(F)AB}_{\hat{\mu}\hat{\nu}} \end{pmatrix} := \mathcal{V} \begin{pmatrix} \mathcal{F}^{(F)}_{\hat{\mu}\hat{\nu}MN} \\ \bar{\mathcal{F}}^{(F)MN}_{\hat{\mu}\hat{\nu}} \end{pmatrix}. \tag{2.7}$$

$H_{\hat{\mu}\hat{\nu}MN}$ is then eliminated from the lagrangean (2.1) by solving

$$\hat{\mathcal{F}}_{\hat{\mu}\hat{\nu}AB} = i\tilde{\hat{\mathcal{F}}}_{\hat{\mu}\hat{\nu}AB} \tag{2.8}$$

where we define the dual by $\tilde{\mathcal{F}}^{\hat{\mu}\hat{\nu}} := (1/2) e^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \mathcal{F}_{\hat{\rho}\hat{\sigma}}$ where $e_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ is the four-dimensional volume element and we follow the convention that $e_{0123} = -e$. In the next subsection we fix the SU(8) symmetry and find expressions for $\tilde{H}_{\hat{\mu}\hat{\nu}MN}$ and $P_{\hat{\mu}ABCD}$.

Finally the theory is invariant under eight supersymmetries. For the fermions the infinitesimal supersymmetry transformations are given by

$$\delta_S \lambda_{ABC} = \left(\sqrt{2}\hat{P}_{\hat{\mu}ABCD}\gamma^{\hat{\mu}}\epsilon^D - \frac{3}{4}\hat{\mathcal{F}}_{[AB}\epsilon_{C]}\right) \tag{2.9}$$

$$\delta_S \psi_{\hat{\mu}A} = \left(\left(\delta_A{}^B D_{\hat{\mu}}(\omega) - Q_{\hat{\mu}A}{}^B\right)\epsilon_B + \frac{1}{4\sqrt{2}}\hat{\mathcal{F}}_{AB}\gamma_{\hat{\mu}}\epsilon_B + O\left(\bar{\lambda}\lambda, \bar{\psi}\lambda\right)\epsilon\right). \tag{2.10}$$

The supersymmetry transformations of the bosonic fields can be found in [7].

2.2 Bertotti-Robinson background

The generic 56 charge black hole solution to $\mathcal{N} = 8$ supergravity can be obtained by applying an $E_{7(7)}$ duality transformation to a 5-parameter generating solution [20]. In particular an arbitrary 56 charge black hole with fixed moduli is obtained by applying an arbitrary element of SU(8)($\subset E_{7(7)}$) to the 5-parameter solution with the same moduli. The 56 charge black hole with arbitrary moduli is then obtained by applying an $E_{7(7)}$ transformation. The 56 "dressed" charges fit into an $E_{7(7)}$ invariant antisymmetric matrix Z_{AB} $(A, B = 1, \ldots, 8)$ transforming in the **28** of SU(8). This matrix can be block diagonalized via an SU(8) transformation as

$$Z_{AB} = \begin{pmatrix} 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_4 & 0 \end{pmatrix}. \tag{2.11}$$

The unbroken global symmetry group of the solution is then given by the subgroup of SU(8) that leaves Z_{AB} invariant. A particular case of interest are the extreme black holes, which can be put in the form (2.11) with $\lambda_2 = \lambda_3 = \lambda_4 = 0$, and therefore have SU(2) $\times$ SU(6) global symmetry group [11]. It follows that any extreme black hole can be obtained from any other extreme black hole by an SU(8) transformation (and then an $E_{7(7)}$ transformation to change the moduli if necessary). Since the mass spectrum is invariant under these transformations then knowing it for one means that we know it for all, and in particular for the intersecting D3-brane solution of [4]. We are actually only interested in the near horizon limit, but clearly the same argument applies. Rather than expanding about the near horizon limit of the intersecting D3-brane solution, which is $AdS_2 \times S^2$ with four non-vanishing vector fields, we instead expand about the same geometry but with only one non-zero vector with flux on S^2 only. We now proceed to describe this background.

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We consider the compactification of the $\mathcal{N}=8$ supergravity theory (2.1) about the BR solution consisting of an $AdS_2 \times S^2$ geometry with curvature tensors

$$\begin{aligned} \overset{\circ}{R}_{\mu\nu\lambda\rho} &= -\frac{1}{l^2}\left(\overset{\circ}{g}_{\mu\lambda}\,\overset{\circ}{g}_{\nu\rho} - \overset{\circ}{g}_{\mu\rho}\,\overset{\circ}{g}_{\nu\lambda}\right), \\ \overset{\circ}{R}_{\alpha\beta\gamma\delta} &= \frac{1}{l^2}\left(\overset{\circ}{g}_{\alpha\gamma}\,\overset{\circ}{g}_{\beta\delta} - \overset{\circ}{g}_{\alpha\delta}\,\overset{\circ}{g}_{\beta\gamma}\right), \end{aligned} \tag{2.12}$$

where $\mu, \nu, \ldots$ are curved AdS_2 indices, $\alpha, \beta, \ldots$ curved S^2 indices, l is the radius of curvature of both spaces, and we put a circle above fields to denote that they are background fields. The only other non-vanishing field that we consider is a two-form flux on S^2 which we take to be[2]

$$\overset{\circ}{G}{}^{[12]}_{\mu\nu} = 0\,, \qquad \overset{\circ}{G}{}^{[12]}_{\alpha\beta} = \frac{1}{l}\,\overset{\circ}{e}_{\alpha\beta}\,. \tag{2.13}$$

i.e. the Freund-Rubin ansatz [21]. The AdS_2 and S^2 volume elements are denoted $e_{\mu\nu}$ and $e_{\alpha\beta}$ respectively and are related to the four-dimensional volume element by $e_{\mu\nu\alpha\beta} = -e_{\mu\nu}e_{\alpha\beta}$. The remaining fields in (2.1) vanish except for the vielbein $\mathcal{V}$ which is equal to the identity

$$\mathcal{V} = \begin{pmatrix} \delta_{[A}{}^{[M}\delta_{B]}{}^{N]} & 0 \\ 0 & \delta^{[A}{}_{[M}\delta^{B]}{}_{N]} \end{pmatrix}. \tag{2.14}$$

That these fields actually solve the equations of motion is clear from the expression (2.18) for $\tilde{H}_{\hat\mu\hat\nu MN}$ given in the next section. When all fields vanish except for $g_{\hat\mu\hat\nu}$ and $B^{[12]}_{\hat\mu}$ and $\mathcal{V}$ is given by (2.14) then

$$\tilde{H}^{MN}_{\hat\mu\hat\nu} = -\delta_{[1}{}^{[M}\delta_{2]}{}^{N]}G^{[12]}_{\hat\mu\hat\nu}\,, \tag{2.15}$$

in which case the action (2.1) reduces to the Einstein-Maxwell action.

The BR solution preserves only two of the eight supersymmetries [10]. To see this set the fermionic supersymmetry transformations (2.9), (2.10) to zero. It immediately follows from $\delta_S\lambda_{ABC} = 0$ that $\epsilon^C = 0$ for $C \neq 1, 2$. The remaining conditions are satisfied trivially except for $\delta_S\psi_{\hat\mu A} = 0$ for $A = 1, 2$ which implies the Killing spinor equations

$$\begin{aligned} D_\mu(\omega)\tilde{\eta}_A - \frac{i}{2l}\rho_\mu\epsilon_{AB}\tilde{\eta}_B &= 0 \\ D_\alpha(\omega)\eta_A - \frac{i}{2l}\rho_5\rho_\alpha\epsilon_{AB}\eta_B &= 0 \end{aligned} \tag{2.16}$$

where we have decomposed ϵ^A into a product of two-component spinors as $\epsilon^A = \tilde{\eta}^A \otimes \eta^A$ and similarly for the γ-matrices as discussed in the appendix.

[2] We put square brackets around the M, N indices when they take specific numerical values in order to avoid confusion with other indices.

2.3 Linearized equations of motion

To find the equations of motion we must solve (2.8) for $\tilde{H}_{\hat\mu\hat\nu MN}$. To do this it is convenient to fix the SU(8) gauge symmetry to the so called symmetric gauge where $\mathcal{V} = \exp(X)$ and

$$X = \begin{pmatrix} 0 & W_{ABCD} \\ \bar{W}^{ABCD} & 0 \end{pmatrix}.$$

W_{ABCD} is complex, completely antisymmetric in A, B, C, D, and satisfies the constraint

$$\bar{W}^{ABCD} = \frac{1}{24}\eta\, \epsilon^{ABCDEFGH} W_{EFGH}\,. \tag{2.17}$$

In this gauge it is straightforward to determine $\tilde{H}_{\hat\mu\hat\nu MN}$ from (2.8). Since our interest is in computing the mass spectrum of the theory about the BR background we only need $\tilde{H}_{\hat\mu\hat\nu MN}$ to quadratic order in fluctuations, therefore it is sufficient to consider

$$\begin{aligned} G^{MN}_{\hat\mu\hat\nu}\tilde{H}^{(B)}_{\hat\mu\hat\nu MN} = &-G^{MN}_{\hat\mu\hat\nu}\left(1 + W + \bar{W} + W^2 + \bar{W}^2\right)_{MNPQ} G^{\hat\mu\hat\nu PQ} + \\ &+ iG^{MN}_{\hat\mu\hat\nu}\left(W - \bar{W} + W^2 - \bar{W}^2\right)_{MNPQ} \tilde{G}^{\hat\mu\hat\nu PQ} \end{aligned} \tag{2.18}$$

for the purely bosonic part of $\tilde{H}_{\hat\mu\hat\nu MN}$, and

$$\begin{aligned} G^{MN}_{\hat\mu\hat\nu}\tilde{H}^{(F)\hat\mu\hat\nu}_{MN} = \frac{1}{\sqrt{2}}\Bigg(& i\bar\psi_{\hat\nu A}\gamma^{[\hat\nu}\, \mathring{\mathcal{F}}_{AB}\gamma^{\hat\mu]}\psi_{\hat\nu B} - \frac{1}{\sqrt{2}} i\bar\psi_{\hat\mu C}\, \mathring{\mathcal{F}}_{AB}\gamma^{\hat\mu}\lambda_{ABC} + \\ & + \frac{i}{72}\eta\, \epsilon^{ABCDEFGH}\bar\lambda_{ABC}\, \mathring{\mathcal{F}}_{DE}\lambda_{FGH}\Bigg) \end{aligned} \tag{2.19}$$

for the fermionic part, where

$$\mathring{\mathcal{F}}_{AB} := \frac{1}{\sqrt{2}}\left(\mathring{G}_{\hat\mu\hat\nu AB} + i\gamma_5 \tilde{\mathring{G}}_{\hat\mu\hat\nu AB}\right)\gamma^{\hat\mu\hat\nu}. \tag{2.20}$$

The solution to (2.8) is not unique. We have used this freedom to simplify the equations of motion as much as possible.

Expressions for $Q_{\hat\mu A}{}^B$ and $P_{\hat\mu ABCD}$ follow straightforwardly in this gauge from (2.3) and (2.17). $Q_{\hat\mu A}{}^B$ is at least linear in W_{ABCD} and therefore can be dropped in the action as it always multiplies a pair of fermions. $P_{\hat\mu ABCD}$ however is given by

$$P_{\hat\mu ABCD} = \partial_{\hat\mu} W_{ABCD} \tag{2.21}$$

and contributes to the quadratic part of the action.

To obtain the linearized equations of motion, substitute (2.18), (2.19) and (2.21) into the action (2.1), expand $g_{\hat\mu\hat\nu}$ and $G^{MN}_{\hat\mu\hat\nu}$ about the background (2.12) and (2.13) as

$$\begin{aligned} g_{\hat\mu\hat\nu} &= \mathring{g}_{\hat\mu\hat\nu} + h_{\hat\mu\hat\nu} \\ G^{MN}_{\hat\mu\hat\nu} &= \mathring{G}^{MN}_{\hat\mu\hat\nu} + g^{MN}_{\hat\mu\hat\nu}\,, \end{aligned} \tag{2.22}$$

vary, and keep only linear terms in the fluctuations in the resulting equations of motion. For the bosonic fields we find

$$\Big(-\frac{1}{2}\nabla^2 h_{\hat\mu\hat\nu}-\frac{1}{2}\nabla_{\hat\mu}\nabla_{\hat\nu}h+\nabla_{(\hat\mu}\nabla^{\hat\alpha}h_{\hat\nu)\hat\alpha}+\overset{\circ}{R}{}^{\hat\alpha}_{(\hat\mu}h_{\hat\nu)\hat\alpha}+\overset{\circ}{R}_{\hat\alpha\hat\mu\hat\nu\hat\beta}h^{\hat\alpha\hat\beta}-$$

$$-\frac{1}{2}h_{\hat\mu\hat\nu}\overset{\circ}{R}-\frac{1}{2}\overset{\circ}{g}_{\hat\mu\hat\nu}\left(-\nabla^2 h+\nabla_{\hat\alpha}\nabla_{\hat\beta}h^{\hat\alpha\hat\beta}-h^{\hat\alpha\hat\beta}\overset{\circ}{R}_{\hat\alpha\hat\beta}\right)\Big)=$$

$$=2\Big(2\overset{\circ}{G}{}^{[12]\hat\alpha}_{(\hat\mu}g^{[12]}_{\hat\nu)\hat\alpha}-\overset{\circ}{G}{}^{[12]}_{\hat\mu\hat\alpha}\overset{\circ}{G}{}^{[12]}_{\hat\nu\hat\beta}h^{\hat\alpha\hat\beta}-$$

$$-\frac{1}{2}\overset{\circ}{g}_{\hat\mu\hat\nu}\left(\overset{\circ}{G}{}^{[12]\hat\alpha\hat\beta}g^{[12]}_{\hat\alpha\hat\beta}-\overset{\circ}{G}{}^{[12]}{}_{\hat\alpha}{}^{\hat\nu}\overset{\circ}{G}{}^{[12]}_{\hat\beta\hat\nu}h^{\hat\alpha\hat\beta}\right)-\frac{1}{4}h_{\hat\mu\hat\nu}\overset{\circ}{G}{}^{[12]\hat\alpha\hat\beta}\overset{\circ}{G}{}^{[12]}_{\hat\alpha\hat\beta}\Big)$$

for the linearized Einstein equations,

$$\nabla_{\hat\mu}\left(g^{[12]\hat\mu\hat\nu}-2\overset{\circ}{G}{}^{[12][\hat\mu}{}_{\hat\alpha}h^{\hat\nu]\hat\alpha}+\frac{1}{2}\overset{\circ}{G}{}^{[12]\hat\mu\hat\nu}h\right)=0\qquad M=1,N=2\tag{2.23}$$

$$\nabla_{\hat\mu}g^{[MN]\hat\mu\hat\nu}=0\qquad M=1,2;N\neq 1,2\tag{2.24}$$

$$\nabla_{\hat\mu}\Big(g^{[MN]\hat\mu\hat\nu}+2\overset{\circ}{G}{}^{[12]\hat\mu\hat\nu}(W+\bar W)_{12MN}-$$

$$-i2\overset{\circ}{\widetilde{G}}{}^{[12]\hat\mu\hat\nu}(W-\bar W)_{12MN}\Big)=0\qquad M,N\neq 1,2\tag{2.25}$$

for the linearized vector equations, and

$$\Big(\nabla^2 W_{12CD}-\left(\overset{\circ}{G}{}^{[12]\hat\mu\hat\nu}+i\overset{\circ}{\widetilde{G}}{}^{[12]\hat\mu\hat\nu}\right)g^{CD}_{\hat\mu\hat\nu}-$$

$$-\overset{\circ}{G}{}^{[12]}_{\hat\mu\hat\nu}\left(\overset{\circ}{G}{}^{[12]\hat\mu\hat\nu}+i\overset{\circ}{\widetilde{G}}{}^{[12]\hat\mu\hat\nu}\right)\bar W^{12CD}\Big)=0\qquad C,D\neq 1,2\tag{2.26}$$

$$\nabla^2 W_{ABCD}=0\qquad A=1,2;B,C,D\neq 1,2\tag{2.27}$$

for the linearized scalar equations. All remaining bosonic equations can be obtained from the above equations by symmetry, complex conjugation, and the constraint (2.17).

The linearized fermion equations of motion follow in a similar way. We find

$$\left(\gamma^{\hat\mu}D_{\hat\mu}\lambda_{12C}+\frac{1}{4}\gamma^{\hat\mu}\overset{\circ}{\mathcal{F}}_{[12]}\psi_{\hat\mu C}\right)=0\qquad C\neq 1,2\tag{2.28}$$

$$\gamma^{\hat\mu}D_{\hat\mu}\lambda_{ABC}=0\qquad A=1,2;B,C\neq 1,2\tag{2.29}$$

$$\left(\gamma^{\hat\mu}D_{\hat\mu}\lambda_{ABC}-\frac{\eta}{12\sqrt{2}}\epsilon^{12ABCFGH}\overset{\circ}{\mathcal{F}}_{[12]}\lambda_{FGH}\right)=0\qquad A,B,C\neq 1,2\tag{2.30}$$

for the spinors and

$$\left(e^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\gamma_{\hat{\sigma}}\gamma_5 D_{\hat{\nu}}\psi_{\hat{\rho}A} + \frac{i}{2\sqrt{2}}\gamma^{[\hat{\nu}}\;\mathring{\mathcal{F}}_{AB}\gamma^{\hat{\mu}]}\psi_{\hat{\nu}B}\right) = 0 \qquad A = 1,2 \tag{2.31}$$

$$\left(e^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\gamma_{\hat{\sigma}}\gamma_5 D_{\hat{\nu}}\psi_{\hat{\rho}A} - \frac{i}{4}\;\mathring{\mathcal{F}}_{[12]}\gamma^{\hat{\mu}}\lambda_{12A}\right) = 0 \qquad A \neq 1,2 \tag{2.32}$$

for the gravitinos.

3. Bosonic masses

3.1 Harmonic expansion on S^2

To find the bosonic mass spectrum on AdS_2 we expand the fields in spherical harmonics on S^2. The expansions are quite simple in this case as all harmonic functions on the 2-sphere can be expressed in terms of just the scalar spherical harmonics Y_{lm}. The expansions of the bosonic fluctuations are then given by (denoting the l, m indices collectively by (k))

$$h_{\mu\nu} = \sum_k H^{(k)}_{\mu\nu} Y_{(k)} \tag{3.1}$$

$$h_{\mu\alpha} = \sum_k \left(B^{(k)}_{1\mu}\nabla_\alpha Y_{(k)} + B^{(k)}_{2\mu} e_{\alpha\beta}\nabla^\beta Y_{(k)}\right) \tag{3.2}$$

$$h_{\alpha\beta} = \sum_k \left(\phi^{(k)}_1 \nabla_\alpha\nabla_\beta Y_{(k)} + \phi^{(k)}_2 e_{(\alpha}{}^{\gamma}\nabla_{\beta)}\nabla_\gamma Y_{(k)} + \phi^{(k)}_3 g_{\alpha\beta} Y_{(k)}\right) \tag{3.3}$$

$$b^{AB}_\mu = \sum_k b^{(k)AB}_\mu Y_{(k)} \tag{3.4}$$

$$b^{AB}_\alpha = \sum_k \left(b^{(k)AB}_1 \nabla_\alpha Y_{(k)} + b^{(k)AB}_2 e_{\alpha\beta}\nabla^\beta Y_{(k)}\right) \tag{3.5}$$

$$W_{ABCD} = \sum_k W^{(k)}_{ABCD} Y_{(k)}\,, \tag{3.6}$$

where the harmonics satisfy[3] $\nabla_\alpha\nabla^\alpha Y_{(k)} = -k(k+1)Y_{(k)}$.

Before substituting the expansions into the linearized equations of motion we can first simplify the expansions by fixing some of the gauge symmetries. Specifically we have four dimensional diffeomorphism invariance and 28 U(1) gauge invariances. To fix the diffeomorphism invariance we work in de Donder-Lorentz gauge

$$\nabla^\alpha h_{\alpha\mu} = 0 = \nabla^\alpha\left(h_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}g^{\gamma\delta}h_{\gamma\delta}\right). \tag{3.7}$$

[3]For the remainder of the paper we set the curvature scale $l = 1$ and drop the $\circ$ symbol above background quantities.

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The U(1) invariances are fixed by the Lorentz-like gauge conditions

$$\nabla^\alpha b_\alpha^{AB} = 0\,. \tag{3.8}$$

Plugging in the above expansions yields the conditions

$$\phi_1^{(k)} = \phi_2^{(k)} = B_{1\mu}^{(k)} = 0 \qquad k > 1 \tag{3.9}$$

$$b_1^{(k)AB} = 0 \qquad k \geq 1\,. \tag{3.10}$$

The gauge fixing conditions do not quite fix all the gauge symmetry, but rather leave diffeomorphism and U(1) gauge symmetries of the zero modes, and further the conformal diffeomorphisms generated by the vectors

$$\xi_\mu = -\nabla_\mu \xi_1^{(1)} Y_{(1)},\, \xi_\alpha = \left(\xi_1^{(1)} \nabla_\alpha Y_{(1)} + \xi_2^{(1)} \overset{\circ}{\epsilon}_{\alpha\beta} \nabla^\beta Y_{(1)}\right). \tag{3.11}$$

The zero mode symmetries we will treat later. The conformal diffeomorphisms are easily dealt with by noting that the $Y_{(1)}$ harmonics satisfy

$$(\nabla_\alpha \nabla_\beta + g_{\alpha\beta}) Y_{(1)} = 0\,. \tag{3.12}$$

It follows that the expansion of $h_{\alpha\beta}$ in the $Y_{(1)}$ sector contains only $\phi_3^{(1)}$ (after a redefinition). Under a conformal diffeomorphism $\phi_3^{(1)}$ transforms as

$$\delta\phi_3^{(1)} = -2\xi_1^{(1)} \tag{3.13}$$

and therefore can be set to zero. Similarly under a conformal diffeomorphism generated by $\xi_2^{(1)}$ we find

$$\delta B_{2\mu}^{(1)} = \nabla_\mu \xi_2^{(1)}\,. \tag{3.14}$$

To fix this symmetry we demand the Lorentz condition

$$\nabla^\mu B_{2\mu}^{(1)} = 0\,. \tag{3.15}$$

3.2 AdS_2 linearized equations of motion

To keep things as simple as possible, we begin by considering only the $\mathcal{N} = 2$ supergravity fluctuations, $h_{\hat{\mu}\hat{\nu}}$ and $b_{\hat{\mu}}^{[12]}$. In terms of the global symmetry group $SU(2) \times SU(6)$ both transform as singlets. Substituting the expansions (3.1)–(3.5) along with the background field strength (2.13) into the linearized equations of motion yields[4]

$$\begin{aligned}
&\Big[\big(\nabla_x^2+2-k(k+1)\big)H^{(k)}_{\mu\nu}-2\nabla_{(\mu}\nabla^\lambda H^{(k)}_{\nu)\lambda}+\\
&+\Big(\nabla_\mu\nabla_\nu-g_{\mu\nu}\big(\nabla_x^2+1-k(k+1)\big)\Big)H^{(k)}+g_{\mu\nu}\nabla^\lambda\nabla^\rho H^{(k)}_{\lambda\rho}+\\
&+2\left(\nabla_\mu\nabla_\nu-g_{\mu\nu}\left(\nabla_x^2-1-\frac{1}{2}k(k+1)\right)\right)\phi_3^{(k)}\Big] = 4g_{\mu\nu}k(k+1)b_2^{(k)} \quad (3.16)\\
&\Big(\big(\nabla_x^2-1-k(k+1)\big)B^{(k)}_{2\mu}-\nabla_\mu\nabla^\nu B^{(k)}_{2\nu}\Big) = 4b_\mu^{(k)[12]} \quad (3.17)\\
&\Big(\nabla^\nu H^{(k)}_{\mu\nu}-\nabla_\mu H^{(k)}-\nabla_\mu\phi_3^{(k)}\Big) = -4\nabla_\mu b_2^{(k)} \quad (3.18)\\
&\Big(\big(\nabla_x^2+4\big)\phi_3^{(k)}+\big(\nabla_x^2-1-k(k+1)\big)H^{(k)}-\nabla^\mu\nabla^\nu H^{(k)}_{\mu\nu}\Big) = 4k(k+1)b_2^{(k)} \quad (3.19)\\
&H^{(k)} = 0 \quad (3.20)\\
&\nabla^\mu B^{(k)}_{2\mu} = 0 \quad (3.21)
\end{aligned}$$

for the Einstein equations and

$$\Big(2\nabla^\mu\nabla_{[\mu}b^{(k)[12]}_{\nu]}-k(k+1)b^{(k)[12]}_\nu-k(k+1)B^{(k)}_{2\nu}\Big) = 0 \quad (3.22)$$

$$\left(\big(\nabla_x^2-k(k+1)\big)b_2^{(k)}+\phi_3^{(k)}-\frac{1}{2}H^{(k)}\right) = 0 \quad (3.23)$$

$$\Big(\nabla^\mu B^{(k)}_{2\mu}+\nabla^\mu b^{(k)[12]}_\mu\Big) = 0 \quad (3.24)$$

for the $M=1, N=2$ vector equations. All equations are valid for $k\geq 2$. The $k=0,1$ cases will be handled separately. Actually one notices that there are more equations than fields. An important consistency check is that the equations are not all independent, as follows easily by taking divergences on the AdS_2 index of the above equations. More generally this follows from the Bianchi identities.

The Einstein equations (3.16), (3.18) and (3.20) allow one to eliminate the metric fluctuation $H^{(k)}_{\mu\nu}$ (locally) in terms of the remaining fields. This follows by a simple counting argument.[5] A homogeneous solution to these equations would have to satisfy $H^{(k)}=0$ and $\nabla^\mu H^{(k)}_{\mu\nu}=0$. These equations are enough to determine $H^{(k)}_{\mu\nu}$ though, i.e. three equations and three unknowns. However $H^{(k)}_{\mu\nu}$ must also satisfy the homogeneous part of (3.16), which clearly cannot be the case in general for arbitrary k. In fact one can show easily that the traceless and divergenceless conditions imply $(\nabla_x^2+2)H^{(k)}_{\mu\nu}=0$, which is only consistent with (3.16) when $k=0$.

The divergence equations (3.21) and (3.24) remove one degree of freedom each from the vectors, reducing them effectively to scalars $(e^{\lambda\rho}\nabla_\lambda B^{(k)}_{2\rho})$ and $(e^{\lambda\rho}\nabla_\lambda b^{(k)[12]}_\rho)$

[4]We use the notation $\nabla_x^2 := g^{\mu\nu}\nabla_\mu\nabla_\nu$.

[5]This argument has also been given recently in [22].

respectively. Acting on the vector equations (3.17) and (3.22) with the operator $e^{\lambda\mu}\nabla_\lambda$ and using the two-dimensional identity

$$2\nabla_{[\mu}a_{\nu]} = -e_{\mu\nu}e^{\lambda\rho}\nabla_\lambda a_\rho \tag{3.25}$$

we obtain the coupled scalar equations

$$\Big(\big(\nabla_x^2 - k(k+1)\big)\, e^{\mu\nu}\nabla_\mu b_\nu^{(k)[12]} - k(k+1)e^{\mu\nu}\nabla_\mu B_{2\nu}^{(k)}\Big) = 0 \tag{3.26}$$

$$\Big(\big(\nabla_x^2 - k(k+1) - 2\big)\, e^{\mu\nu}\nabla_\mu B_{2\nu}^{(k)} - 4e^{\mu\nu}\nabla_\mu b_\nu^{(k)[12]}\Big) = 0\,. \tag{3.27}$$

Similarly substituting the Einstein equations (3.18) and (3.20) into the $\phi_3^{(k)}, b_2^{(k)}$ equations of motion (3.19) and (3.23) we obtain the coupled equations

$$\Big(\big(\nabla_x^2 - k(k+1) - 2\big)\, \phi_3^{(k)} + 4k(k+1)b_2^{(k)}\Big) = 0 \tag{3.28}$$

$$\Big(\big(\nabla_x^2 - k(k+1)\big)\, b_2^{(k)} + \phi_3^{(k)}\Big) = 0\,. \tag{3.29}$$

Defining the scalar mass by $(\nabla_x^2 - m^2)\phi = 0$, the respective mass matrices are easily diagonalized and we find

$$m^2 = k(k-1), (k^2+3k+2) \tag{3.30}$$

in each case, where each mass at level $k \geq 2$ is $(2k+1)$ degenerate.

$k = 1$ **sector.** The $k = 1$ sector equations of motion follow from the $k \geq 2$ equations after taking into account the gauge fixing condition $\phi_3^{(1)} = 0$ and the relations (3.12). The latter equations imply that the Einstein equations (3.19) and (3.20) are not separate equations but rather are replaced by

$$\big((\nabla_x^2 - 2)H^{(1)} - \nabla^\mu\nabla^\nu H_{\mu\nu}^{(1)}\big) = 8b_2^{(1)}\,. \tag{3.31}$$

Furthermore (3.21) no longer follows from the Einstein equations but nevertheless holds due to our choice of gauge (3.15). As a consequence the coupled equations (3.27) for $(e^{\mu\nu}\nabla_\mu B_{2\nu}^{(1)})$ and $(e^{\mu\nu}\nabla_\mu b_\nu^{(1)[12]})$ continue to hold for $k = 1$, and therefore also the masses (3.30). However at $k = 1$ the mass $m^2 = k(k-1)$ vanishes and therefore one of the vectors must be massless, in particular one can show that

$$(\nabla_x^2 + 1)\left(B_{2\mu}^{(1)} - \frac{2}{\sqrt{2}}b_\mu^{(1)[12]}\right) = 0\,. \tag{3.32}$$

This field can be gauged away locally due to the residual gauge transformations (3.14) of $B_{2\mu}^{(1)}$, i.e. $\delta B_{2\mu}^{(1)} = \nabla_\mu\xi_2^{(1)}$ with $\nabla_x^2\xi_2^{(1)} = 0$ implies

$$(\nabla_x^2 + 1)\nabla_\mu\xi_2^{(1)} = 0\,. \tag{3.33}$$

Although this field can be removed locally, it still has boundary degrees of freedom and will be important in filling a representation of SU$(1,1|2)$ discussed later, so we shall continue to treat it as a $k=1$ field.

By the same argument as in the $k \geq 2$ case above, $H^{(1)}_{\mu\nu}$ can be eliminated in terms of $b^{(1)}_2$. Specifically the trace of (3.16) implies that $H^{(1)} = 8b^{(1)}_2$, which along with (3.16) and (3.18) is enough to eliminate $H^{(1)}_{\mu\nu}$. Substituting for $H^{(1)}$ in (3.23) results in

$$(\nabla^2_x - 6)b^{(1)}_2 = 0\,. \tag{3.34}$$

It follows that the mass spectrum (3.30) holds for $k=1$ for the scalars $b^{(1)}_2$ and $\phi^{(1)}_3$ for the mass $m^2 = (k^2+3k+2)$, but not for $m^2 = k(k-1)$.

$k=0$ sector. In the zero mode sector $k=0$ the equations of motion are given by (3.16), (3.19), and (3.22). In this sector though we still have two-dimensional diffeomorphism invariance and a U(1) gauge symmetry. Before fixing these symmetries note that the trace of (3.16) implies

$$\left(\nabla^2_x - 2\right)\phi^{(0)}_3 = 0 \tag{3.35}$$

completing one of the $m^2 = (k^2+3k+2)$ towers above to $k=0$.

The remaining fields can be gauged away locally. This can be seen by first fixing the diffeomorphism symmetry by demanding

$$\nabla^\mu H^{(k)}_{\mu\nu} = 5\nabla_\nu \phi^{(0)}_3\,. \tag{3.36}$$

This implies from (3.19) that

$$(\nabla^2_x - 1)\left(\phi^{(0)}_3 - \frac{1}{4}H^{(0)}\right) = 0\,. \tag{3.37}$$

Residual diffeomorphisms satisfy

$$(\nabla^2_x \xi_\nu + \nabla_\nu\nabla^\mu\xi_\mu - \xi_\nu) = 0 \tag{3.38}$$

and therefore $(\nabla^2_x - 1)\nabla^\mu\xi_\mu = 0$ allowing us to further set $H^{(0)} = 4\phi^{(0)}_3$. To finally eliminate any independent degrees of freedom of $H^{(0)}_{\mu\nu}$ consider a solution to the homogeneous part of it's equations of motion. It must be traceless, divergenceless, and satisfy

$$(\nabla^2_x + 2)H^{(0)}_{\mu\nu} = 0 \tag{3.39}$$

from (3.16). The remaining residual diffeomorphisms however satisfy

$$(\nabla^2_x + 2)\nabla_{(\mu}\xi_{\nu)} = 0 \tag{3.40}$$

and $\nabla^\mu\xi_\mu = 0$, and can be used to eliminate $H^{(0)}_{\mu\nu}$ in terms of $\phi^{(0)}_3$.

Fields	$\mathrm{SU}(2) \times \mathrm{SU}(6)$	modes	mass	conformal dimension	degeneracy
$\phi_3^{(k)}$	$(\mathbf{1}, \mathbf{1})$	$k \geq 1$	$k(k-1)$	k	$2k+1$
$b_2^{(k)}$	$(\mathbf{1}, \mathbf{1})$	$k \geq 0$	$k^2 + 3k + 2$	$k+2$	$2k+1$
$e^{\mu\nu}\nabla_\mu b_\nu^{(k)[12]}$	$(\mathbf{1}, \mathbf{1})$	$k \geq 2$	$k(k-1)$	k	$2k+1$
$e^{\mu\nu}\nabla_\mu B_{2\nu}^{(k)}$	$(\mathbf{1}, \mathbf{1})$	$k \geq 1$	$k^2 + 3k + 2$	$k+2$	$2k+1$
W_{ABCD} $A{=}1,2; B,C,D{\neq}1,2$	$(\mathbf{2}, \mathbf{20})$	$k \geq 0$	$k(k+1)$	$k+1$	$2k+1$
$\mathcal{F}_{MN}^{(k)}$ $M = 1,2; N \neq 1,2$	$(\mathbf{2}, \mathbf{6})$	$k \geq 1$	$k(k+1)$	$k+1$	$2k+1$
$W_{12CD}^{(k)}; C, D \neq 1,2$	$(\mathbf{1}, \mathbf{15})$	$k \geq 1$	$k(k-1)$	k	$2k+1$
$\mathcal{F}_{CD}^{(k)}; C, D \neq 1,2$	$(\mathbf{1}, \mathbf{15})$	$k \geq 0$	$k^2 + 3k + 2$	$k+2$	$2k+1$

Table 1: Scalar fields.

Finally the gauge field $b_\mu^{(0)[12]}$ can be eliminated by the U(1) gauge symmetry exactly as described above. A summary of the masses of the $\mathcal{N} = 2$ supergravity scalars is given in table 1. A final comment before leaving this section is to note that for $k \geq 2$ half of the local degrees of freedom of $B_{2\mu}^{(k)}$ and all local degrees of freedom of $H_{\mu\nu}^{(k)}$ (in the $k \geq 1$ case as well) were eliminated by equations of motion. Normally these are eliminated by gauge symmetries as in the $k = 0$ case above. In fact though their elimination for $k \geq 2$ also follows by gauge symmetry. Four-dimensional diffeomorphism invariance must remove eight "four"-dimensional fields. Four of these fields were removed immediately when we fixed the gauge (3.10). The remaining four followed via the equations of motion.

3.3 Completion of bosonic fields to $\mathcal{N} = 8$ supergravity

The remaining bosonic fields of $\mathcal{N} = 8$ supergravity are the scalars W_{ABCD} and vectors $b_{\hat\mu}^{MN}$ for $M, N \neq 1, 2$. Before computing their mass spectrum let's first understand the $\mathrm{SU}(2) \times \mathrm{SU}(6)$ transformation properties of the fields. For the scalars this follows trivially from their SU(8) transformation properties. For the vectors it is actually the field strength tensors (2.5) that transform under SU(8). Substituting the vector and scalar expansions (3.4), (3.5) and (3.6) and the expression for the bosonic part of $H_{\hat\mu\hat\nu MN}$ in the symmetric gauge (2.18) we find

$$\mathcal{F}_{MN}^{(k)} := \left(e^{\lambda\rho}\nabla_\lambda b_\rho^{(k)MN} + ik(k+1)b_2^{(k)MN} + i2\left(W_{12MN}^{(k)} + \bar{W}^{(k)12MN}\right)\right), \quad (3.41)$$

where $\mathcal{F}_{\mu\nu MN} = -e_{\mu\nu}/\sqrt{2}\sum_k \mathcal{F}_{MN}^{(k)} Y_{(k)}$. The fields $\mathcal{F}_{MN}^{(k)}$ therefore transform in the

28 of SU(8) and it's SU(2) $\times$ SU(6) transformation properties follow. Note that the scalar contribution to (3.41) vanishes when either M or N is 1 or 2.

To compute the remaining bosonic masses let's begin with the $A = 1,2$, $B,C,D \neq 1,2$ scalars W_{ABCD} and the $M = 1,2$, $N \neq 1,2$ vectors $g_{\hat{\mu}\hat{\nu}}^{MN}$, which are already completely decoupled from all other fields. Substituting the harmonic expansions (3.4), (3.5,) and (3.6) into the equations (2.24) and (2.27) yields

$$
\begin{aligned}
\left(\nabla_x^2 - k(k+1)\right) W_{ABCD}^{(k)} &= 0\,, \qquad k \geq 0 \\
\left(2\nabla^\mu \nabla_{[\mu} b_{\nu]}^{(k)MN} - k(k+1) b_\nu^{(k)MN}\right) &= 0\,, \qquad k \geq 0 \\
\nabla^\mu b_\mu^{(k)MN} &= 0\,, \qquad k \geq 1 \qquad (3.42) \\
\left(\nabla_x^2 - k(k+1)\right) b_2^{(k)MN} &= 0\,, \qquad k \geq 1\,. \qquad (3.43)
\end{aligned}
$$

As before the two equations for the vector $b_\mu^{(k)MN}$ are equivalent to

$$
\left(\nabla_x^2 - k(k+1)\right) e^{\lambda\rho} \nabla_\lambda b_\rho^{(k)MN} = 0, \qquad k \geq 1\,. \tag{3.44}
$$

Furthermore we can combine the equations for $e^{\lambda\rho}\nabla_\lambda b_\rho^{(k)MN}$ and $b_2^{(k)MN}$ into the SU(8) covariant equation

$$
(\nabla_x^2 - k(k+1))\mathcal{F}_{MN}^{(k)} = 0, \qquad k \geq 1\,. \tag{3.45}
$$

The zero mode of the vector $b_\mu^{(0)MN}$ can be completely eliminated as discussed before leaving us with two towers of complex scalars of mass $m^2 = k(k+1)$ and degeneracy $(2k+1)$ for fixed group indices. The $A = 1,2$, $B,C,D \neq 1,2$ scalars $W_{ABCD}^{(k)}$ transform in the $(\mathbf{2},\mathbf{20})$ of SU(2) $\times$ SU(6) and the $M = 1,2$, $N \neq 1,2$ scalars $\mathcal{F}_{MN}^{(k)}$ in the $(\mathbf{2},\mathbf{6})$ representation.

Substituting the harmonic expansions (3.4), (3.5) and (3.6) into the remaining bosonic equations of motion (2.25) and (2.26) for the $M,N \neq 1,2$ vectors $g_{\hat{\mu}\hat{\nu}}^{MN}$ and the $C,D \neq 1,2$ scalars W_{12CD} we obtain

$$
\begin{aligned}
&\Big(\left(\nabla_x^2 - k(k+1)\right)W_{12CD}^{(k)} - 2\bar{W}^{(k)12CD} + \\
&\qquad + i\left(e^{\mu\nu}\nabla_\mu b_\nu^{(k)CD} + ik(k+1) b_2^{(k)CD}\right)\Big) = 0\,, \qquad k \geq 0 \\
&\Big(2\nabla^\mu \nabla_{[\mu} b_{\nu]}^{(k)MN} - k(k+1) b_\nu^{(k)MN} + \\
&\qquad + i2e_{\mu\nu}\nabla^\mu\left(W_{12MN}^{(k)} - \bar{W}^{(k)12MN}\right)\Big) = 0\,, \qquad k \geq 0 \\
&\nabla^\mu b_\mu^{(k)MN} = 0\,, \qquad k \geq 1 \\
&\left(\left(\nabla_x^2 - k(k+1)\right) b_2^{(k)MN} - 2\left(W_{12MN}^{(k)} + \bar{W}^{(k)12MN}\right)\right) = 0\,, \qquad k \geq 1\,. \qquad (3.46)
\end{aligned}
$$

As before the equations for the vectors $b_\mu^{(k)MN}$ may be rewritten as

$$\Big(\big(\nabla_x^2 - k(k+1)\big)\, e^{\mu\nu}\nabla_\mu b_\nu^{(k)MN} - i2\nabla_x^2 \Big(W^{(k)}_{12MN} - \bar{W}^{(k)12MN}\Big)\Big) = 0\,, \qquad k \geq 1\,. \tag{3.47}$$

After some rearranging the equations can be rewritten in $\mathrm{SU}(2)\times\mathrm{SU}(6)$ covariant form as

$$\begin{aligned}\big(\nabla_x^2 - k(k+1) + 2\big)\, W^{(k)}_{12CD} + i\mathcal{F}^{(k)}_{CD} &= 0, \qquad k \geq 1\\ \big(\nabla_x^2 - k(k+1) - 4\big)\, \mathcal{F}^{(k)}_{CD} - i4(k(k+1)-2)W^{(k)}_{12CD} &= 0, \qquad k \geq 1\,.\end{aligned} \tag{3.48}$$

Diagonalizing the mass matrix produces two towers of complex scalars with masses

$$m^2 = k(k-1), k^2 + 3k + 2\,, \qquad k \geq 1 \tag{3.49}$$

and with level k degeneracy $(2k+1)$ for fixed group indices. The zero mode of the vector $b_\mu^{(0)MN}$ can be eliminated as described before leaving us with one massive scalar to complete the tower $m^2 = k^2 + 3k + 2$ to $k = 0$. Both towers of scalars transform in the $(\mathbf{1}, \mathbf{15})$ of $\mathrm{SU}(2)\times\mathrm{SU}(6)$. All bosonic masses as well as degeneracies and $\mathrm{SU}(2)\times\mathrm{SU}(6)$ transformation properties are tabulated in table 1.

4. Fermionic masses

4.1 Spinor harmonic expansion on S^2

A complete set of complex two-component spinors on S^2 is given by ψ_{slm} and $\rho_5\psi_{slm}$ [23] (γ-matrix conventions are given in the appendix) where $s = \pm$, $m = 0, \ldots, (l+1)$, and

$$\begin{aligned}(\rho^\alpha D_\alpha - i(l+1))\psi_{slm} &= 0\\ (\rho^\alpha D_\alpha + i(l+1))\rho_5\psi_{slm} &= 0\,.\end{aligned} \tag{4.1}$$

The spinors also satisfy the complex conjugation property

$$(\psi_{slm})^* = i(s)\rho_5\psi_{-s,lm}\,. \tag{4.2}$$

The 4-dimensional spinor and gravitino expansions therefore take the form

$$\lambda_{ABC} = \sum \Big(\lambda^{(s,k)}_{+ABC} \otimes \psi_{(s,k)} + \lambda^{(s,k)}_{-ABC} \otimes \rho_5\psi_{(s,k)}\Big) \tag{4.3}$$

$$\psi_{\mu A} = \sum \Big(\psi^{(s,k)}_{+\mu A} \otimes \psi_{(s,k)} + \psi^{(s,k)}_{-\mu A} \otimes \rho_5\psi_{(s,k)}\Big) \tag{4.4}$$

$$\begin{aligned}\psi_{\alpha A} = \sum \Big(&\psi^{(s,k)}_{+A} \otimes D_{(\alpha)}\psi_{(s,k)} + \chi^{(s,k)}_{+A} \otimes \rho_\alpha\psi_{(s,k)} +\\ &+ \psi^{(s,k)}_{-A} \otimes D_{(\alpha)}(\rho_5\psi_{(s,k)}) + \chi^{(s,k)}_{-A} \otimes \rho_\alpha\rho_5\psi_{(s,k)}\Big)\end{aligned} \tag{4.5}$$

where $D_{(\alpha)} := (D_\alpha - (1/2)\rho_\alpha\rho^\beta D_\beta)$ and we have again combined the l, m indices into k. The Majorana condition gives rise to

$$(\psi_+^{(s,k)})^* = -i(s)\psi_-^{(-s,k)} \tag{4.6}$$

for all $+/-$ fermionic coefficients in the above expansions.

The linearized supersymmetry transformations for the gravitini take the form

$$\delta_s\psi_{\mu A} = \left(D_\mu\epsilon_A - \frac{i}{2}\rho_\mu \otimes 1\, \epsilon_{AB}\epsilon_B\right), \tag{4.7}$$

$$\delta_s\psi_{\alpha A} = \left(D_\alpha\epsilon_A - \frac{i}{2}1 \otimes \rho_5\rho_\alpha\epsilon_{AB}\epsilon_B\right) \tag{4.8}$$

for $A = 1, 2$ and

$$\delta_s\psi_{\hat\mu A} = D_{\hat\mu}\epsilon_A\,, \qquad A \neq 1, 2\,. \tag{4.9}$$

Expanding ϵ_A as in (4.3) it is easy to see after using the identity $D_\alpha = D_{(\alpha)} + (1/2)\rho_\alpha\rho^\beta D_\beta$ that $\chi_{\pm A}^{(s,k)}$ can be removed for $k \geq 1, A = 1, 2$ and $k \geq 0$, $A \neq 1, 2$. The case $k = 0$, $A = 1, 2$ is slightly more subtle because of the existence of the Killing spinors $\psi_{(s,0)}$ which satisfy

$$D_\alpha\psi_{(s,0)} = \frac{i}{2}\rho_\alpha\psi_{(s,0)}\,. \tag{4.10}$$

The variations of $\chi_{\pm A}^{(s,0)}$ under supersymmetry transformations for $A = 1, 2$ are easily shown to be

$$\delta_s\chi_{+A}^{(s,0)} = \frac{i}{2}\left(\epsilon_{+A}^{(s,0)} + \epsilon_{AB}\epsilon_{-B}^{(s,0)}\right) \tag{4.11}$$

$$\delta_s\chi_{-A}^{(s,0)} = \frac{i}{2}\epsilon_{AB}\left(\epsilon_{+B}^{(s,0)} + \epsilon_{BC}\epsilon_{-C}^{(s,0)}\right) \tag{4.12}$$

and therefore that $(\chi_{+A}^{(s,0)} + \epsilon_{AB}\chi_{-B}^{(s,0)})$ is invariant and cannot be gauged away. The opposite combination $(\chi_{+A}^{(s,0)} - \epsilon_{AB}\chi_{-B}^{(s,0)})$ however can be gauged away. The $\psi_{\alpha A}$ expansions then simplify to

$$\begin{aligned}\psi_{\alpha A} = \sum_{k\geq 1}\Big(\psi_{+A}^{(s,k)} \otimes D_{(\alpha)}\psi_{(s,k)} + \psi_{-A}^{(s,k)} \otimes D_{(\alpha)}(\rho_5\psi_{(s,k)})\Big) + \\ + \left(\chi_{+A}^{(s,0)} \otimes \rho_\alpha\psi_{(s,0)} + \epsilon_{AB}\chi_{+B}^{(s,0)} \otimes \rho_\alpha\rho_5\psi_{(s,0)}\right)\end{aligned} \tag{4.13}$$

for $A = 1, 2$ and

$$\psi_{\alpha A} = \sum_{k\geq 1}\left(\psi_{+A}^{(s,k)} \otimes D_{(\alpha)}\psi_{(s,k)} + \psi_{-A}^{(s,k)} \otimes D_{(\alpha)}(\rho_5\psi_{(s,k)})\right) \tag{4.14}$$

for $A \neq 1, 2$.

4.2 $\mathcal{N}=2$ Fermi field content

As in the bosonic case we again begin by finding the mass spectrum for the $\mathcal{N}=2$ fermi fields only. This consists of the two gravitini $\psi_{\hat{\mu}A}$ for $A=1,2$. Substituting the expansions (4.4), (4.13) into the linearized equations of motion (2.31) we obtain

$$\left(\frac{i}{2}\left((k+1)^2-1\right)\rho_\mu\psi_{+A}^{(s,k)}-(k+1)\psi_{-\mu A}^{(s,k)}+\epsilon_{AB}\psi_{+\mu B}^{(s,k)}\right)=0 \tag{4.15}$$

$$\left(-\rho^\mu\psi_{+\mu A}^{(s,k)}+\rho^\mu D_\mu\psi_{+A}^{(s,k)}-i\epsilon_{AB}\psi_{+B}^{(s,k)}\right)=0 \tag{4.16}$$

$$\left(-\rho_4 e^{\mu\nu}D_\mu\psi_{-\nu A}^{(s,k)}-\frac{i}{2}(k+1)\rho^\mu D_\mu\psi_{+A}^{(s,k)}-\frac{1}{2}(k+1)\epsilon_{AB}\psi_{+B}^{(s,k)}\right)=0 \tag{4.17}$$

for the $k\geq 1$ modes, the zero modes we handle separately.

Once again there are more equations than fields, however one equation is not linearly independent. The first equation, along with it's complex conjugate, eliminates $\psi_{\pm\mu A}^{(s,k)}$ in terms of $\psi_{\pm A}^{(s,k)}$. After some trivial rearranging we find

$$\left(\rho^\mu D_\mu+(k+1)\right)\eta_A^{(s,k)}=0\,, \tag{4.18}$$

where

$$\eta_A^{(s,k)}:=\left(\psi_{+A}^{(s,k)}-i\psi_{-A}^{(s,k)}\right) \tag{4.19}$$

Defining AdS_2 spinor masses by $m=|\kappa|$ for $(\rho^\mu D_\mu-\kappa)\lambda=0$ we find two towers of complex spinors, i.e. $A=1,2$, each with mass $m=(k+1)$ for $k\geq 1$ and degeneracy $2(k+1)$. The two sets of spinors can be further decomposed into two $(\mathbf{2},\mathbf{1})$ representations of $\mathrm{SU}(2)\times\mathrm{SU}(6)$.

$k=0$ sector. The equations of motion in the $l=0$ sector are

$$\begin{aligned}\left(i2\rho_4 e^{\mu\nu}D_\nu\chi_{+A}^{(s,0)}+\rho^\mu\epsilon_{AB}\chi_{+B}^{(s,0)}+e^{\mu\nu}\rho_4\left(\psi_{+\nu A}^{(s,0)}+\epsilon_{AB}\psi_{-\nu B}^{(s,0)}\right)\right)&=0\\ \left(\rho^\mu D_\mu\chi_{+A}^{(s,0)}-i\epsilon_{AB}\chi_{+B}^{(s,0)}-\rho_4 e^{\mu\nu}D_\mu\psi_{-\nu A}^{(s,0)}-\frac{i}{2}\rho^\mu\psi_{+\mu A}^{(s,0)}\right)&=0\,.\end{aligned} \tag{4.20}$$

The first equation along with it's complex conjugate are enough to show that

$$(\rho^\mu D_\mu+1)(\delta_{AB}+i\epsilon_{AB})\chi_{+B}^{(s,0)}=0 \tag{4.21}$$

and

$$\psi_{+\mu A}^{(s,0)}=-\epsilon_{AB}\psi_{-\mu B}^{(s,0)}. \tag{4.22}$$

Therefore we have a single complex spinor with mass $m=1$ transforming under the $(\mathbf{2},\mathbf{1})$ representation of $\mathrm{SU}(2)\times\mathrm{SU}(6)$. We also have the two-dimensional gravitino $\psi^{(s,0)}_{+\mu A}$. This field however can be gauged away by using the residual supersymmetry transformations satisfying $\epsilon^{(s,0)}_{+A} = -\epsilon_{AB}\epsilon^{(s,0)}_{-B}$. Specifically one can first demand

$$\rho^\mu \psi^{(s,0)}_{+\mu A} = -4\epsilon_{AB}\chi^{(s,0)}_{+B}\,. \tag{4.23}$$

The equation of motion (4.20) then reduces to $e^{\mu\nu}D_\mu\psi^{(s,0)}_{+\nu A} = 0$, which is also the equation satisfied by residual supersymmetry transformations $\delta_s\psi^{(s,0)}_{+\mu A}$ so that $\psi^{(s,0)}_{+\mu A}$ can be removed locally.

4.3 Completion of the fermionic fields to $\mathcal{N} = 8$

The remaining fermionic fields are the spinors λ_{ABC} and the $A \neq 1,2$ gravitini $\psi_{\hat{\mu}A}$. The $A = 1,2; B,C \neq 1,2$ spinors are already completely decoupled so their masses follow almost immediately. Substituting the expansion (4.3) into the equation of motion (2.29) we find after taking appropriate linear combinations

$$\big(\rho^\mu D_\mu - (k+1)\big)\xi^{(s,k)}_{ABC} = 0\,, \tag{4.24}$$

where

$$\xi^{(s,k)}_{ABC} := \left(\lambda^{(s,k)}_{+ABC} + i\lambda^{(s,k)}_{-ABC}\right). \tag{4.25}$$

Therefore we have one tower of complex spinors with mass $m = (k+1)$, degeneracy $2(k+1)$ at level $k \geq 0$, and transforming in the $(\mathbf{2},\mathbf{15})$ of SU(2) $\times$ SU(6).

The $A,B,C \neq 1,2$ spinor masses are also straightforward to compute. Substituting the expansion (4.3) as well as the background field strength (2.13) into the equation of motion (2.30) yields after some rearranging[6]

$$\begin{aligned}\big(\rho^\mu D_\mu - k\big)\left(\xi^{(s,k)}_{ABC} + i\frac{\eta}{6}\epsilon^{12ABCFGH}\xi^{(s,k)}_{FGH}\right) &= 0\\ \big(\rho^\mu D_\mu - (k+2)\big)\left(\xi^{(s,k)}_{ABC} - i\frac{\eta}{6}\epsilon^{12ABCFGH}\xi^{(s,k)}_{FGH}\right) &= 0\,.\end{aligned} \tag{4.26}$$

We therefore have two towers of complex spinors with masses $m = k$ and $m = (k+2)$ respectively. Both towers have degeneracy $2(k+1)$ at level $k \geq 0$ and transform in the $(\mathbf{1},\mathbf{20})$ of SU(2) $\times$ SU(6).

[6] These expressions may appear incorrect at first sight because of the mixing of up and down SU(8) indices. The $\xi^{(s,k)}_{ABC}$ fields however do not transform covariantly under SU(8) because of the γ_5 matrix appearing in the transformation law of λ_{ABC}. Their transformation law is easy to find and indeed one may show that an SU(6) transformation maps these equations into themselves and their complex conjugates (after letting $s \to -s$).

The remaining coupled fermions are the λ_{12A} spinors and the $A \neq 1,2$ gravitini $\psi_{\hat{\mu}A}$. Substituting the expansions (4.3), (4.4) and (4.5) into the equations of motion (2.28) and (2.32) gives rise to

$$\left(\rho^\mu D_\mu \lambda^{(s,k)}_{+12A} - i(k+1)\lambda^{(s,k)}_{-12A} - \frac{i}{\sqrt{2}}\rho^\mu \psi^{(s,k)}_{-\mu C}\right) = 0 \tag{4.27}$$

$$\left(\frac{i}{2}((k+1)^2-1)\rho_\mu \psi^{(s,k)}_{+A} - (k+1)\psi^{(s,k)}_{-\mu A} + \frac{1}{\sqrt{2}}\rho_\mu \lambda^{(s,k)}_{-12A}\right) = 0 \tag{4.28}$$

$$\left(\rho^\mu D_\mu \psi^{(s,k)}_{+A} - \rho^\mu \psi^{(s,k)}_{+\mu A}\right) = 0 \tag{4.29}$$

$$\left(-\rho_4 e^{\mu\nu} D_\mu \psi^{(s,k)}_{-\nu A} - \frac{i}{2}(k+1)\rho^\mu D_\mu \psi^{(s,k)}_{+A} + \frac{i}{\sqrt{2}}\lambda^{(s,k)}_{+12A}\right) = 0 \tag{4.30}$$

for $k \geq 1$. The second equation eliminates $\psi^{(s,k)}_{-\mu A}$ in terms of the other fields (and similarly $\psi^{(s,k)}_{+\mu A}$ after complex conjugation). One of the remaining equations is redundant, while the others imply after taking appropriate linear combinations

$$(\rho^\mu D_\mu - k)\left(\eta^{(s,k)}_A + \frac{\sqrt{2}}{k+2}\xi^{(s,k)}_{12A}\right) = 0 \tag{4.31}$$

$$(\rho^\mu D_\mu - (k+2))\left(\eta^{(s,k)}_A - \frac{\sqrt{2}}{k}\xi^{(s,k)}_{12A}\right) = 0\,, \tag{4.32}$$

where $\xi^{(s,k)}_{ABC}$ is defined in (4.25) and $\eta^{(s,k)}_A$ in (4.19).

For the zero modes the equations are given by (4.27), and (4.28) with $k=0$ and $\psi^{(s,0)}_{\pm A} = 0$. It follows that $\psi^{(s,k)}_{-\mu A}$ can be eliminated and that

$$\left(\rho^\mu D_\mu - 2\right)\xi^{(s,0)}_{12A} = 0\,. \tag{4.33}$$

For these fields we therefore find two towers of complex spinors, one with mass $m = k$ for $k \geq 1$ and the other with mass $m = (k+2)$ for $k \geq 0$. Both have degeneracy $2(k+1)$ at level k and transform in the $(\mathbf{1},\mathbf{6})$ of $\mathrm{SU}(2) \times \mathrm{SU}(6)$. The complete set of fermion masses are summarized in table 2.

5. SU(1, 1|2) multiplets

Irreducible representations of $\mathrm{SU}(1,1|2)$ are labelled by the eigenvalues of the L_0 and J_0 generators of the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ subalgebra. The irreducible representations have been constructed in [24] and in particular the so-called short multiplets which

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Fields	SU(2) × SU(6)	modes	mass	conformal dimension	degeneracy
$\eta_A^{(s,k)}$ $A=1,2$	$(\mathbf{2},\mathbf{1})$	$k \geq 1$	$k+1$	$k+\frac{3}{2}$	$2(k+1)$
	$(\mathbf{2},\mathbf{1})$	$k \geq 0$	$k+1$	$k+\frac{3}{2}$	$2(k+1)$
$\xi_{ABC}^{(s,k)}$ $A=1,2; B,C\neq 1,2$	$(\mathbf{2},\mathbf{15})$	$k \geq 0$	$k+1$	$k+\frac{3}{2}$	$2(k+1)$
$\xi_{ABC}^{(s,k)}$ $A,B,C\neq 1,2$	$(\mathbf{1},\mathbf{20})$	$k \geq 0$	k	$k+\frac{1}{2}$	$2(k+1)$
	$(\mathbf{1},\mathbf{20})$	$k \geq 0$	$k+2$	$k+\frac{5}{2}$	$2(k+1)$
$\xi_{12C}^{(s,k)}; C\neq 1,2$	$(\mathbf{1},\mathbf{6})$	$k \geq 1$	k	$k+\frac{1}{2}$	$2(k+1)$
$\eta_C^{(s,k)}; C\neq 1,2$	$(\mathbf{1},\mathbf{6})$	$k \geq 0$	$k+2$	$k+\frac{5}{2}$	$2(k+1)$

Table 2: Spinor fields.

will be of interest here. The short multiplet irreducible representations consist of the states

$$D^{(k)}(k) \oplus 2D^{(k-1/2)}\left(k+\frac{1}{2}\right) \oplus D^{(k-1)}(k+1) \tag{5.1}$$

for half-integer $k \geq 1/2$ (where in the $k=1/2$ case it is understood that the $D^{(-1/2)}(3/2)$ states are missing). We now proceed to show that the states listed in tables 1 and 2 fill various short multiplet representations of $\mathrm{SU}(1,1|2)$.

An irreducible representation of SU(2) labelled by J_0 eigenvalue j has $(2j+1)$ states. For the harmonic expansions of the scalar fields (with degeneracy $(2k+1)$) this implies $j=k$. For the spinors with degeneracy $2(k+1)$ we have $j=(k+1/2)$. The L_0 eigenvalue of the states comes from the AdS/CFT map developed in [8, 9]. The conformal weight of a boundary conformal field corresponding to an AdS_2 scalar was shown to be

$$h_{scalar} = \frac{1}{2}\left(1+\sqrt{1+4m^2}\right), \tag{5.2}$$

and for a spinor [25]

$$h_{spinor} = m + \frac{1}{2}\,. \tag{5.3}$$

Plugging in the various scalar and spinor masses that we have found results in the conformal weights shown in tables 1 and 2 respectively.

The $\mathcal{N}=8$ supergravity fields also carry $\mathrm{SU}(2)\times\mathrm{SU}(6)$ indices, and therefore so will the boundary states. We therefore label the complete boundary states as

$$D^{(j)}(h)(\mathbf{R_2}\times\mathbf{R_6})\,, \tag{5.4}$$

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where as above h and j label the $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SU}(2)$ content of $\mathrm{SU}(1,1|2)$ and $\mathbf{R_2}\times\mathbf{R_6}$ labels the $\mathrm{SU}(2)\times\mathrm{SU}(6)$ content. From tables 1 and 2 we can now read off the representations. For the $\mathcal{N}=2$ sector we find two sets of

$$D^{(k)}(k)(\mathbf{1},\mathbf{1})\oplus D^{\left(k-\frac{1}{2}\right)}\left(k+\frac{1}{2}\right)(\mathbf{2},\mathbf{1})\oplus D^{(k-1)}(k+1)(\mathbf{1},\mathbf{1}) \tag{5.5}$$

for $k\geq 1$ and $k\geq 2$. For the remaining fields we find the representations

$$\begin{aligned}
D^{(k+1/2)}\left(k+\frac{1}{2}\right)(\mathbf{1},\mathbf{20})\oplus D^{(k)}(k+1)(\mathbf{2},\mathbf{20})\oplus D^{(k-1/2)}\left(k+\frac{3}{2}\right)(\mathbf{1},\mathbf{20})\,, &\quad k\geq 0\\
D^{(k+1/2)}\left(k+\frac{1}{2}\right)(\mathbf{1},\mathbf{6})\oplus D^{(k)}(k+1)(\mathbf{2},\mathbf{6})\oplus D^{(k-1/2)}\left(k+\frac{3}{2}\right)(\mathbf{1},\mathbf{6})\,, &\quad k\geq 1\\
D^{(k)}(k)(\mathbf{1},\mathbf{15})\oplus D^{(k-1/2)}\left(k+\frac{1}{2}\right)(\mathbf{2},\mathbf{15})\oplus D^{(k-1)}(k+1)(\mathbf{1},\mathbf{15})\,, &\quad k\geq 1\,.
\end{aligned}$$

6. Conclusions

We have computed the mass spectrum of the $\mathcal{N}=8$ supergravity theory about $AdS_2\times S^2$. We have shown that the corresponding spectrum of states in the boundary conformal field theory lies in short multiplets of the AdS_2 supergroup $\mathrm{SU}(1,1|2)$. The states further carry $\mathrm{SU}(2)\times\mathrm{SU}(6)$ indices inherited from the spontaneously broken $\mathrm{SU}(8)$ gauge group of the supergravity theory.

Eventually one hopes to identify these boundary states in the $d=1$, $\mathcal{N}=4$ n-particle Calogero model, conjectured [3] to be the dual conformal quantum mechanics model to the the near horizon region of the intersecting D3-brane solution [4]. As noted in [3] though, this theory has yet to be constructed. Recently however the one-dimensional single particle Calogero model has been constructed [6] for an arbitrary number of supersymmetries. Generalization of the model to arbitrary n should be straightforward. Once this is done a detailed check on the correspondence between the two theories should be possible.

We have ignored the boundary degrees of freedom except for one massless vector in the $\mathcal{N}=2$ sector which was needed to fill an $\mathrm{SU}(1,1|2)$ representation. Such fields may also be necessary in realizing the AdS/CFT duality. It should be straightforward to extract this information from the results in this paper.

While this work was being written up another paper [22] appeared reporting similar results for the $\mathcal{N}=2$ sector of the $\mathcal{N}=8$ supergravity theory discussed here. [22] also considered in detail the boundary degrees of freedom for $\mathcal{N}=2$ supergravity.

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A. Gamma Matrices

Our gamma matrix and spinor conventions are

$$
\begin{aligned}
\{\gamma^a, \gamma^b\} &= 2\eta^{ab}\,, & (\gamma^a)^\dagger &= \gamma^0\gamma^a\gamma^0\,,\\
(\gamma^a)^a &= \gamma^a\,, & (\gamma^a)^T &= \gamma^0\gamma^a\gamma^0\,,\\
\gamma_5 &= i\gamma^0\gamma^1\gamma^2\gamma^3\,, & \{\gamma_5, \gamma^a\} &= 0\,,\\
\gamma^\mu &= e^\mu{}_a\gamma^a\,, & \bar{\lambda} &= \lambda^\dagger\gamma^0\,,\\
\gamma^{[a_1a_2\cdots a_n]} &= \gamma^{[a_1}\gamma^{a_2}\ldots\gamma^{a_n]}
\end{aligned}
\tag{A.1}
$$

where the metric signature is $(-,+,+,+)$ and a and μ both take values in $0,1,2,3$. By choosing the gamma matrices to be real, the Majorana condition reduces to $\lambda^* = \lambda$. We split the γ^a-matrices under $\mathrm{SO}(1,3) \to \mathrm{SO}(1,1) \otimes \mathrm{SO}(2)$ as

$$
\gamma^a = \rho^a \otimes \rho_5\,, \qquad \gamma^i = 1 \otimes \rho^i\,,
\tag{A.2}
$$

where now $a = 0,1$ and $i = 2,3$. We define the $\mathrm{SO}(1,1)$ and $\mathrm{SO}(2)$ "γ_5" matrices by $\rho_4 := \rho^0\rho^1$ and $\rho_5 := i\rho^2\rho^3$ respectively. One possible choice of ρ-matrices satisfying the above γ-matrix conventions is given by

$$
\begin{aligned}
\rho^0 &= \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, & \rho^1 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\\
\rho^2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \rho^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\end{aligned}
\tag{A.3}
$$

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